Math 221: Test 1 Review Sheet Answers

Problem Solving:

1. First note that the last page of the book is $4n$ since there are 4 pages per sheet. So, the sum of the page numbers on each side of a sheet is $4n+1$. There are n sheets, so there are $2n$ sides. Thus, the sum of the page numbers is $2n(4n+1)$. When we did this in class, we looked at the 2 sheet and 3 sheet cases, so we solved a simpler problem.
2. Consider the following:

$$\begin{array}{cccccccccc} & 1 & + & 2 & + & 3 & + & ... & + & n \\ + & n & + & ... & + & 3 & + & 2 & + & 1 \\ \hline \\ & (n+1) & + & (n+1) & + & (n+1) & + & .... & + & (n+1) & = n(n+1) \end{array}$$

   We added up the numbers twice, so $1+2+3+...+n = \frac{n(n+1)}{2}$.
3. Consider the following:

$$\begin{array}{cccccccccc} & 2 & + & 4 & + & 6 & + & ... & + & 40 \\ + & 40 & + & 38 & + & 36 & + & ... & + & 2 \\ \hline \\ & 42 & + & 42 & + & 42 & + & .... & + & 42 & = 20(42) \end{array}$$

   We added up the numbers twice, so $2+4+6+...+40 = \frac{20(42)}{2} = 420$.
4. Consider the simpler case of 4 people, A, B, C, and D. First, A shakes 3 hands: B, C, and D. Next, B shakes 2 hands: C and D. Finally, C shakes D's hand. So, the number of handshakes is $3+2+1=6$. Now, consider the simpler case of 5 people, A, B, C, D and E. First, A shakes 4 hands: B, C, D and E. Next, B shakes 3 hands: C, D, and E. Next, C shakes 2 hands: D and E. Finally, D shakes E's hand. So, the number of handshakes is $4+3+2+1=10$. So, it appears that for 10, the number of handshakes would be $9+8+7+6+5+4+3+2+1 = \frac{9(10)}{2} = 45$.
5. Let's make a table. A * denotes that the person came that day.

Day:	0	1	2	3	4	5	6
Maid:	*		*		*		*
Gardener:	*			*			*

   So, it took 6 days for the maid and the gardener to come on the same day again.
6. You start on Floor 1, then move up 3 floors to Floor 4, down 2 floors to Floor 2, up 7 floors to Floor 9, down 5 floors to Floor 4, and then up 7 floors to stop at the top floor, Floor 11. You can also think of this as $1+3-2+7-5+7=11$.
7. Guess and check: The answer comes out to be 3 and 14.
8. The sum of your top 5 quiz grades plus 30, the divided by the number of quizzes, 6, is your quiz average, 80. Consider the following equation:

$$\frac{(\text{sum of top 5 quiz grades}) + 30}{6} = 80$$
$$(\text{sum of top 5 quiz grades}) + 30 = 480$$
$$(\text{sum of top 5 quiz grades}) = 450$$

   Now, to find the average, take the sum of your top 5 quiz grades and divide it by 5. So, your new average is 90.
9. One way to do this is to draw the 24 heads, then attach 2 feet to each. This gives 48 feet used. Now, add two more feet to each head until you run out of feet. You have 32 more feet to give, so that makes 16 heads with 4 feet and 8 heads with 2 feet. So there are 16 pigs and 8 chickens.

Another way to do this is to solve the equations $x + y = 24$ and $4x + 2y = 80$ simultaneously, where x is the number of pigs and y is the number of chickens. Using the first equation, we solve for y and get $y = 24 - x$. Next, we substitute this into the second equation to get:

$$\begin{aligned} 4x + 2(24 - x) &= 80 \\ 4x + 48 - 2x &= 80 \\ 2x + 48 &= 80 \\ 2x &= 32 \\ x &= 16 \end{aligned}$$

Now, $y = 24 - x = 24 - 16 = 8$. So there are 16 pigs and 8 chickens.

10. First, notice that the sum of the numbers 1 through 9 is 45, so as there are 3 row sums, there will be sums of 15 in each row, column, and main diagonal. Now, write out the ways to sum 3 numbers to 15: $1+5+9$, $2+5+8$, $3+5+7$, $4+5+6$, $2+4+9$, $3+4+9$, $1+6+8$, $2+6+7$. Since 5 is the only number to show up in 4 sums, it must be in the middle. Next, the numbers 2, 4, 6, and 8 show up in 3 sums, so these must be the corners. However, 2 cannot be paired with 8 in a row or column since $2 + 5 + 8 = 15$ and 5 is in the middle, so they must be diagonally opposite. Fill in the other two corners, and then fill in the remaining spaces so that the sums are where there is one blank in a row or column. The rest work by your logic. There are several possible answers, but one is

2	9	4
7	5	3
6	1	8

11. Let n be your number. Then we are performing the following operation to n:

$$\begin{aligned} \frac{(n+15) \times 4 - 8}{4} - 12 &= \frac{4n + 60 - 8}{4} - 12 \\ &= \frac{4n + 52}{4} - 12 \\ &= n + 13 - 12 \\ &= n + 1 \end{aligned}$$

So, your professor is merely subtracting 1 from the number.

12. It is clear that there are 64 1×1 squares since the checkerboard is 8×8. Now, looking at the 2×2 squares, 7 can be found in the bottom row and 7 more along the left column, so there are 49 of these. Looking at the 3×3 squares, 6 can be found in the bottom row and 6 more along the left column, so there are 36 of these. Similarly, there are 25 4×4 squares, 16 5×5 squares, 9 6×6 squares, 4 7×7 squares, and 1 8×8 square (the whole checkerboard). So, the total number of squares is $64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = 204$.

13. Michael and Travis did not come in first place, so Andrew came in first place. Travis beat Michael, so Travis came in second place and Michael came in third place.

14. Since there is only 1 winner in a game of checkers, you'll need to play at least 6 games for two people to have won 3 games each.

15. Danny arrived at 10:10 PM, so nobody else could have arrived at that time. So, since Heather arrived 15 minutes before Michael and Michael could not have arrived at 10:10 PM, Heather must have arrived at 10:00 PM and Michael arrived at 10:15 PM. As everyone else arrived at the other times, Andrew arrived at 9:55 PM.

Patterns

1. We are adding 5 each time, so the next term is 23.
2. We are adding a number that is going up by 2 each time, $(0+3=3,\ 3+5=8,\ 8+7=15)$, so the next term is $15+9=24$.
3. The next figure is a large hexagon formed with sides composed of 4 smaller hexagons.
4. The next figure is a 4×4 square.

Base 10 Numbers

1. Exchange 10 units for 1 long, giving you 11 longs and 6 units. Next, exchange 10 longs for a flat, giving you 13 flats and 1 long. Next, exchange 10 flats for 1 block, giving you 2 blocks and 3 flats. 2316_{ten}
2. Exchange 20 units for 2 longs, giving you 10 longs and 6 units. Next, exchange 10 longs for a flat, giving you 8 flats and 0 longs. 806_{ten}
3. Exchange 30 units for 3 longs, giving you 21 longs and 4 units. Next, exchange 20 longs for 2 flats, giving you 10 flats and 1 long. Next, exchange 10 flats for 1 block, giving you 3 blocks and 0 flats. 3014_{ten}

Base 5 Numbers

1. Exchange 5 units for 1 long, giving you 9 longs and 2 units. Next, exchange 5 longs for 1 flat, giving you 5 flats and 4 longs. Next, exchange 5 flats for a block, giving you 2 blocks and 0 flats. 2042_{five}
2. Exchange 15 units for 3 longs, giving you 11 longs and 4 units. Next, exchange 10 longs for 2 flats, giving you 9 flats and 1 long. Next, exchange 5 flats for 1 block, giving you 1 block and 4 flats. 1414_{five}
3. Exchange 30 units for 6 longs, giving you 20 longs and 0 units. Next, exchange 20 longs for 4 flats, giving you 10 flats and 0 longs. Next, exchange 10 flats for 2 blocks, giving you 4 blocks and 0 flats. 4000_{five}

Converting Base 5 to Base 10

1. $3\cdot 5^2+0\cdot 5+4=75+0+4=79_{\text{ten}}$
2. $1\cdot 5^3+3\cdot 5^2+4\cdot 5+2=125+75+20+2=222_{\text{ten}}$
3. $2\cdot 5^3+1\cdot 5^2+4\cdot 5+0=250+25+20+0=295_{\text{ten}}$
4. $4\cdot 5^3+3\cdot 5^2+2\cdot 5+1=500+75+10+1=586_{\text{ten}}$

Converting Base 10 to Base 5

1. $400/125=\underline{3}\text{ R }25\Longrightarrow 25/25=\underline{1}\text{ R }0\Rightarrow 0/5=\underline{0}\text{ R }\underline{0}$: 3100_{five}
2. $161/125=\underline{1}\text{ R }36\Longrightarrow 36/25=\underline{1}\text{ R }11\Rightarrow 11/5=\underline{2}\text{ R }\underline{1}$: 1121_{five}
3. $486/125=\underline{3}\text{ R }111\Longrightarrow 111/25=\underline{4}\text{ R }11\Rightarrow 11/5=\underline{2}\text{ R }\underline{1}$: 3421_{five}
4. $276/125=\underline{2}\text{ R }26\Longrightarrow 26/25=\underline{1}\text{ R }1\Rightarrow 1/5=\underline{0}\text{ R }\underline{1}$: 2101_{five}
5. $502/125=\underline{4}\text{ R }2\Longrightarrow 2/25=\underline{0}\text{ R }2\Rightarrow 2/5=\underline{0}\ 2\ \underline{1}$: 4002_{five}
6. $256/125=\underline{2}\text{ R }6\Longrightarrow 6/25=\underline{0}\text{ R }6\Rightarrow 6/5=\underline{1}\text{ R }\underline{1}$: 2011_{five}
7. $219/125=\underline{1}\text{ R }94\Longrightarrow 94/25=\underline{3}\text{ R }19\Rightarrow 19/5=\underline{3}\text{ R }\underline{4}$: 1334_{five}
8. $26/25=\underline{1}\text{ R }1\Longrightarrow 1/5=\underline{0}\text{ R }\underline{1}$: 101_{five}

Counting Base 5 Numbers

1_{five}, 2_{five}, 3_{five}, 4_{five}, 10_{five}, 11_{five}, 12_{five}, 13_{five}, 14_{five}, 20_{five}, 21_{five}, 22_{five}, 23_{five}, 24_{five}, 30_{five}, 31_{five}, 32_{five}, 33_{five}, 34_{five}, 40_{five}, 41_{five}, 42_{five}, 43_{five}, 44_{five}, 100_{five}, 101_{five}, 102_{five}, 103_{five}, 104_{five}, 110_{five}, 111_{five}, 112_{five}, 113_{five}, 114_{five}, 120_{five}, 121_{five}, 122_{five}, 123_{five}, 124_{five}, 130_{five}, 131_{five}, 132_{five}, 133_{five}, 134_{five}, 140_{five}, 141_{five}, 142_{five}, 143_{five}, 144_{five}, 200_{five}, 201_{five}, 202_{five}, 203_{five}, 204_{five}, 210_{five}, 211_{five}, 212_{five}, 213_{five}, 214_{five}, 220_{five}, 221_{five}, 222_{five}, 223_{five}, 224_{five}, 230_{five}, 231_{five}, 232_{five}, 233_{five}, 234_{five}, 240_{five}, 241_{five}, 242_{five}, 243_{five}, 244_{five}, 300_{five}, 301_{five}, 302_{five}, 303_{five}, 304_{five}, 310_{five}, 311_{five}, 312_{five}, 313_{five}, 314_{five}, 320_{five}, 321_{five}, 322_{five}, 323_{five}, 324_{five}, 330_{five}, 331_{five}, 332_{five}, 333_{five}, 334_{five}, 340_{five}, 341_{five}, 342_{five}, 343_{five}, 344_{five}, 400_{five}, 401_{five}, 402_{five}, 403_{five},

404_{five}, 410_{five}, 411_{five}, 412_{five}, 413_{five}, 414_{five}, 420_{five}, 421_{five}, 422_{five}, 423_{five}, 424_{five}, 430_{five}, 431_{five}, 432_{five}, 433_{five}, 434_{five}, 440_{five}, 441_{five}, 442_{five}, 443_{five}, 444_{five}, 1000_{five}.

Set Builder Notation

1. $\{x \mid x \text{ is a negative even integer}\}$ or $\{-2x \mid x \in \mathbb{N}\}$
2. Let $n! = n \cdot (n-1) \cdot ... \cdot 3 \cdot 2 \cdot 1$. So $6 = 3!$, $24 = 4!$, etc. Then this set is $\{1!, 2!, 3!, 4!, 5!, ...\}$, which can be written as $\{x! \mid x \in \mathbb{N}\}$.
3. $\{x \mid x \text{ is an odd multiple of } 3\}$ or $\{3x \mid x \in \mathbb{N} \text{ and } x \text{ is odd}\}$
4. $\{x \mid x \text{ is a real number greater than or equal to 3 and less than } \pi\}$ or $\{x \in \mathbb{R} \mid 0 \leq x < \pi\}$
5. Note that this is the same as $\{0, 1, 4, 9, 16, ...\}$, which can be written as $\{x^2 \mid x \in \mathbb{Z} \text{ and } x \geq 0\}$
6. $\{\{x\} \mid x \in \mathbb{R}\}$

Cardinal Numbers

1. 50
2. 5
3. ∞
4. 0
5. ∞
6. 5

Complements

1. $\overline{A} = \{1, 3, 5, 7, 9\}$
2. $\overline{B} = \varnothing$
3. $\overline{C} = \{2, 4, 5, 6, 8, 9, 10\}$

Elements and Subsets

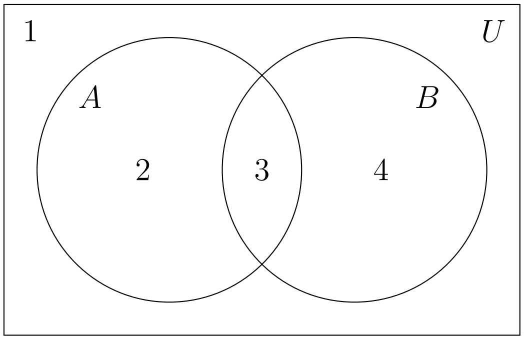
1. $\{0, 2, 4, 6\} \not\subseteq \{0, 2, 4\}$
2. $0 \notin \{x^2 \mid x \in \mathbb{N}\}$
3. $\varnothing \subseteq \varnothing$ (further, $=$)
4. $\varnothing \subseteq \{0, 2, 4, 6\}$ (further, $\subset$)
5. $\sqrt{-1} \notin \mathbb{R}$ (it is imaginary, so not a real number)
6. $\{2^n \mid n \in \mathbb{N}\} \subseteq \{2n \mid n \in \mathbb{N}\}$ (further, $\subset$ since 6 is not in the first set)
7. $\mathbb{N} \subseteq \mathbb{N}$ (further, $=$)
8. $\{0, 2, 4, 6\} \subseteq \{2x \mid x \in \mathbb{Z}\}$ (further, $\subset$)
9. $\{0, 2, 4, 6\} \not\subseteq \varnothing$
10. $8 \in \{0, 2, 4, 6, ...\}$
11. $\{0, 2, 4\} \subseteq \{0, 2, 4, 6\}$ (further, $\subset$)
12. $-1 \notin \mathbb{N}$
13. $0 \in \{0, 2, 4, 6\}$
14. $\varnothing \subseteq \{\varnothing\}$ (further, $\subset$, but can also be $\in$)
15. $-2 \not\subseteq \varnothing$
16. $8 \notin \{0, 2, 4, 6\}$
17. $\{0, 2, 4, 6\} \not\subseteq \mathbb{N}$ (0 is not in $\mathbb{N}$)
18. $5 \in \mathbb{Z}$
19. $\{0, 2, 7\} \subseteq \{0, 2, 7\}$ (further, $=$)
20. $\mathbb{Z} \subseteq \mathbb{R}$ (further, $\subset$)

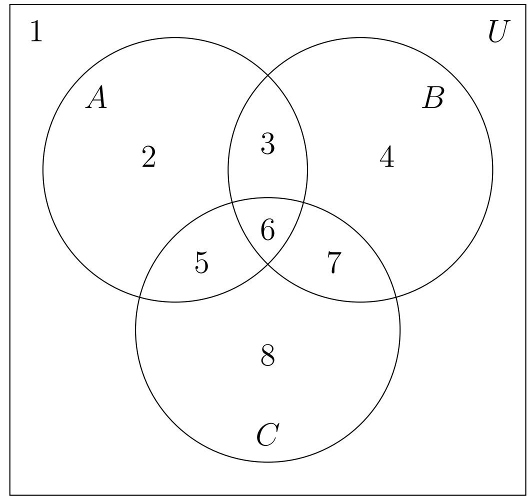
Unions, Intersections, and Set Differences

1. $A \cap C = \{2, 4, 6\}$
2. $A \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
3. $D - A = \{1, 3, 5, 7, 9\}$
4. $D \cup \varnothing = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
5. $B \cap A = \varnothing$
6. $\overline{A} \cap B = \{1, 3, 5\}$
7. $C - \overline{B} = \{3, 5\}$
8. $\overline{D} \cap A = \varnothing$
9. $A - \mathbb{Z} = \varnothing$
10. $\varnothing \cup \varnothing = \varnothing$
11. $C \cap \mathbb{R} = \{2, 3, 4, 5, 6, 7\}$
12. $\varnothing - A = \varnothing$

Venn Diagrams

For these, I will give the regions to shade by numbers based on the following diagrams.





1. Shade Region 3.
2. Shade Regions 2-4.
3. Shade Region 2.
4. Shade Regions 1 and 4.
5. Shade Regions 1, 3, and 4.
6. Shade Region 1.
7. Shade Region 3.
8. Shade Region 1.
9. Shade Regions 1, 2, and 4.
10. Shade Regions 2-8.
11. Shade Regions 2, 3, 5, 6, and 7.
12. Shade Regions 3-7.
13. Shade Regions 2, 3, and 5.
14. Shade Region 2.
15. Shade Region 1.